

S.NO: 22N1-PM

Course Code: PGME1

A.D.M.COLLEGE FOR WOMEN, NAGAPATTINAM

(AUTONOMOUS)

M. Sc. (Mathematics) Degree Examination

I Semester – November 2022

EC I – ADVANCED NUMERICAL ANALYSIS

Time: 3 hours

Maximum Marks: 75

Section –A

10X2=20

Answer ALL the Questions:

1. Define rate of convergence.
2. State Sturm's theorem.
3. State Cayley – Hamilton Theorem.
4. Define spectral radius of a matrix.
5. Define spline function.
6. Write Lagrange Bivariate interpolation polynomial.
7. Write the techniques for finding numerical differentiation.
8. Write composite Simpson's rule.
9. Write the formula for 2nd order implicit R.K Method.
10. Define Discretization error.

Section -B

5X5=25

Answer **ALL** the Questions:

11. a) Find a real root of the equation $x^3 - 4x - 9 = 0$ using bisection method.

(or)

b) Explain Birge-Vieta method.

12. a) Using the Jacobi method find all the eigen values and the

corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$

(or)

b) Transform the matrix $M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ to the tridiagonal form by

Given's method using exact arithmetic.

13. a) Find Newton's bivariate interpolating polynomial from the following data

y\x	0	1	2
0	1	3	7
1	3	6	11
2	7	11	17

(or)

b) Obtain a linear polynomial approximation to the function $f(x) = x^3$ on $[0, 1]$ using the least square approximation with $W(x) = 1$.

14. a) Find the Jacobian matrix for the system of equations

$$f_1(x,y) = x^2 + y^2 - x = 0 \text{ and } f_2(x,y) = x^2 - y^2 - y = 0 \text{ at the point } (1, 1).$$

(or)

b) Evaluate $I = \int_0^1 dx/(1+x)$ using Gauss Legendre three-point formula.

15. a) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$ using the fourth order classical RK method.

(or)

b) Given the initial value problem $u' = -2tu^2$, $u(0) = 1$. Estimate $u(0.4)$ using modified Euler Cauchy method with $h = 0.2$.

Section -C

3 X 10 = 30

16. Find the number of real and complex roots of the polynomial equation

$$P_3(x) = x^3 - 5x + 1 = 0.$$

17. Determine the largest eigenvalue and the corresponding eigenvector of

the matrix $\mathbf{A} = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$ and correct it to 3 decimal places using power method.

18. Given the following values of $f(x)$ and $f'(x)$. Estimate the value of $f(0.5)$

and $f(-0.5)$ using Hermite Interpolation. The exact values are $f(-0.5) = \frac{33}{64}$

and $f(0.5) = \frac{97}{64}$

x	-1	0	1
f(x)	1	1	3
f'(x)	1	3	7

19. (i) Write short note on Richardson's extrapolation method.

(ii) Explain the Lobatto integration method.

20. Use the Euler method to solve numerically the initial value problem

$$u' = -2tu^2, u(0) = 1 \text{ with } h = 0.2, 0.1 \text{ and } 0.05 \text{ on the interval } [0, 1].$$

Determine the bound for the error. Apply Richardson's extrapolation to improve the computed value $u(0.1)$.
